Journal of Engineering Physics and Thermophysics, Vol. 72, No. 4, 1999

CRITICAL ANGLE OF REGULAR INTERACTION UPON COLLISION OF TWO UNIDENTICAL STRONG SKEW SHOCK WAVES IN A GAS WITH A CONSTANT POLYTROPIC INDEX

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UDC 534.2

In the present work on the basis of relationships of the hydrodynamic theory of shock waves, the author determines a critical angle of existence of a regular collision regime (RCR) for two unidentical plane skew shock waves (SSW) in a gas, the change in the polytropic index of which behind the shock wave front can be neglected.

Upon collision of two strong SSW in a polytropic gas the existence of a regime of interaction for small impact angles is admitted [1], when a reflected shock wave exists in the wake of each incident shock wave (Fig. 1), and when on the projection of the contact surface (SC) onto a plane perpendicular to the fronts of both SSW, there is a point of convergence of four shock waves (O). Let us assume that the index H corresponds to a stronger SSW and the index L, to a weaker SSW, $D_H > D_L >> c$, $a = D_H > D_L$. In a coordinate system related to the point O we have:

$$\frac{D_H}{\sin \varphi_H} = \frac{D_L}{\sin \varphi_L}, \quad \varphi = \varphi_L + \varphi_H, \tag{1}$$

whence

$$\varphi_H = \arctan \frac{a \sin \varphi}{1 + a \cos \varphi}, \quad \varphi_L = \arctan \frac{\sin \varphi}{a \cos \varphi}, \tag{2}$$

$\varphi_H > \varphi_L$, since $a > 1$. The state of a gas ahead of SW1 and SW2 (Fig. 1) is characterized by the parameters

$$\rho_{0,H} = \rho_{0,L} = \rho, \quad p_{0,H} = p_{0,L} = p, \quad q_{0,H} = q_{0,L} = q = \frac{D_H}{\sin \varphi_{1,H}} = \frac{D_L}{\sin \varphi_{1,L}} \tag{3}$$

and by the polytropic index k, which is identical for the entire volume. The state of a medium in the wake of discontinuity surfaces can be determined by means of a system of equations

$$p_{i,j} = \frac{2\rho_{i-1,j}\, q_{i-1,j}^2}{k+1} \sin^2 (\varphi_{i,j} + s_{i,j} \vartheta_{i-1,j}) - \frac{k-1}{k+1} p_{i-1,j},$$

$$K_{i,j} = \frac{k + 1 + (k-1)\dfrac{p_{i,j}}{p_{i-1,j}}}{k - 1 + (k+1)\dfrac{p_{i,j}}{p_{i-1,j}}}, \quad \rho_{i,j} = \frac{\rho_{i-1,j}}{K_{i,j}}, \tag{4}$$

$$q_{i,j} = q_{i-1,j} \cos (\varphi_{i,j} + s_{i,j} \vartheta_{i-1,j}) \sqrt{1 + \tan^2 (\varphi_{i,j} + s_{i,j} \vartheta_{i-1,j})\, K_{i,j}^2},$$

Scientific-Research Institute of Pulse Processes of the Belarusian State Scientific and Production Concern of Powder Metallurgy, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 4, pp. 697-699, July-August, 1999. Original article submitted September 18, 1998.

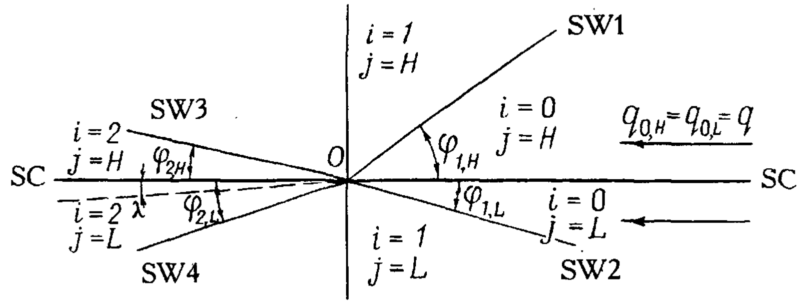

Fig. 1. Diagram of interaction of two SSW in an RCR.

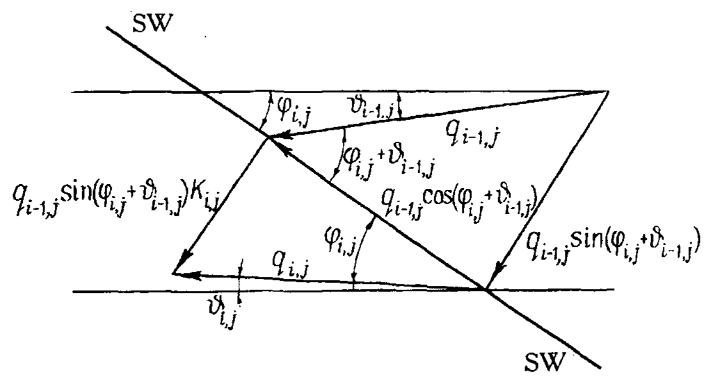

Fig. 2. Diagram of change in the gas flow velocity vector on its passage through the discontinuity surface.

$$\vartheta_{i,j} = \varphi_{i,j} - \arccos\sqrt{\left(\frac{1}{1 + \tan^2(\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})\, K_{i,j}^2}\right)}, \quad s_{i,j} = \begin{cases} 1, & i = 2 \\ 0, & i \neq 2 \end{cases},$$

with the corresponding change in i and j, $i = 1, 2$; $j = H, L$ (a diagram of the expansion of the gas flow velocity vector on its passage through the discontinuity surface is given in Fig. 2). System (4) at all the values of i and j contains 16 equations for determining 18 unknowns: $\rho_{1,H}, \rho_{2,H}, \rho_{1,L}, \rho_{2,L}, p_{1,H}, p_{2,H}, p_{1,L}, p_{2,L}, q_{1,H}, q_{2,H}, q_{1,L}, q_{2,L}, \vartheta_{1,H}, \vartheta_{2,H}, \vartheta_{1,L}, \vartheta_{2,L}, \varphi_{2,H}, \varphi_{2,L}$. This system is closed by equilibrium conditions that require the equality of a pressure and normal velocity component over both sides of the SC:

$$p_{2,H} = p_{2,L}, \quad q_{2,H}\sin\vartheta_{2,H} = q_{2,L}\sin\vartheta_{2,L}. \tag{5}$$

Substitution of the values into the first equality of (5) gives the correlation between $\varphi_{2,L}$ and $\varphi_{2,H}$

$$\sin^2(\varphi_{2,H} + \vartheta_{1,H}) = \frac{\rho_{1,L}}{\rho_{1,H}}\left(\frac{q_{1,L}}{q_{1,H}}\right)^2 \sin^2(\varphi_{2,L} + \vartheta_{1,L}) + (k-1)\frac{p_{1,L} - p_{1,H}}{2\rho_{1,H}q_{1,H}^2}. \tag{6}$$

The second relation of (5) allows us to obtain a transcendental equation for the determination of $\varphi_{2,L}$ and subsequent calculation of all the quantities that characterize the originating gas flow. However, within the framework of the present work it is not presupposed to carry out complete analysis of system (4). In order to find critical angles of existence for an RCR, it is sufficient to restrict ourselves to analysis of Eq. (6). In fact, the correlation between $\varphi_{2,L}$ and $\varphi_{2,H}$ within the region of real angles can be established if

$$\frac{\rho_{1,L}}{\rho_{1,H}}\left(\frac{q_{1,L}}{q_{1,H}}\right)^2 \sin^2(\varphi_{2,H} + \vartheta_{1,H}) + (k-1)\frac{p_{1,L} - p_{1,H}}{2\rho_{1,H}q_{1,H}^2} \geq 0. \tag{7}$$

We note that since $D_H > D_L$, $p_{1,H} > p_{1,L}$. Inequality (7) can be transformed to the form of

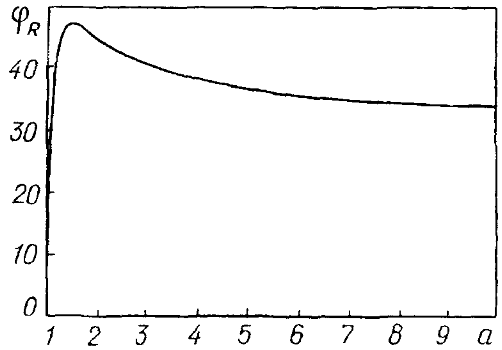

Fig. 3. Dependence of the critical angle of regular collision on α.

$$\sin^2(\varphi_{2,H} + \vartheta_{1,H}) \leq \frac{\rho_{1,H}}{\rho_{1,L}} \left(\frac{q_{1,H}}{q_{1,L}}\right)^2 \left[1 - (k-1)(\sin^2\varphi_{1,H} - \sin^2\varphi_{1,L}) \frac{\rho q^2}{\rho_{1,H} q_{1,H}^2}\right], \tag{8}$$

which is satisfied for the real values of the angles only in the case when

$$(k-1)(\sin^2\varphi_{1,H} - \sin^2\varphi_{1,L}) \frac{\rho q^2}{\rho_{1,H} q_{1,H}^2} \leq 1 . \tag{9}$$

Performing subsequent transformations of relation (9), by determining the maximum possible angle $\varphi_{1,H}$ which corresponds in it to a sign of equality, and introducing the notation

$$K_0 = \frac{2k}{k-1} \frac{p}{\rho D_H^2} + 1 \quad \text{and} \quad K = \frac{k-1}{k+1} K_0 , \tag{10}$$

we obtain

$$\frac{2}{a^2} \frac{(a^2 - 1)\sin^2\varphi_{1,H}}{K_0 \sqrt{1 - \sin^2\varphi_{1,H}(1 - K^2)}} = 1 . \tag{11}$$

By solving Eq. (11) only for the real angles, we write

$$\sin^2\varphi_{1,H} = \left(\frac{a^2}{a^2 - 1}\right)^2 (1 - K^2) \frac{K_0}{8} \left[\sqrt{\left(1 + \left(\frac{a^2 - 1}{a^2}\right)^2 \frac{16}{K_0^2 (1 - K^2)^2}\right)} - 1\right] = g . \tag{12}$$

It turns out that it is more convenient to introduce the notation

$$b = \tan \arcsin \sqrt{g} \tag{13}$$

and, using the first equation of system (2), to pass to the total angle φ. The critical angle of the regular regime of interaction will be given here in the form

$$\varphi_R = \arcsin \left[\frac{b}{a(1 + b^2)} \left(1 + \sqrt{\left(a^2 - \frac{b^2}{1 + b^2}\right)}\right)\right] . \tag{14}$$

As already noted we implement an RCR only for $\varphi \leq \varphi_R$. Figure 3 illustrates the dependence $\varphi_R(a)$ calculated initially for air ($\rho = 1.29$ kg/m^3, $k = 1.4$) at $D_H = 1000$ m/sec. This curve depends only slightly on the parameters of the gas and shock waves with the exception of a. The calculations carried out for air, krypton ($\rho = 3.733$ kg/m^3, $k = 5/3$), methane (0.717 and 4/3), and helium (0.1785 and 5/3) gave virtually identical results. There are no reasons to suppose that these values for other gases can change essentially. At very large a ($a >> 1$) φ_R for all the gases tends to $\sim 29.3^o$. The maximum value of $\varphi_R \approx 48.8^o$ is attained at $a \approx 1.555$.

It should be noted that for small a ($a < 1.5$) the value of φ_R decreases sharply and vanishes already at $a \approx 1.117$, thus ensuring the absence of an RCR for all the smaller values of a. In particular, this is valid also at $a = 1$; only initial condition (5) looks somewhat different:

$$p_{2,H} = p_{2,L} = 0\,,\ q_{2,H} \sin \vartheta_{2,H} = q_{2,L} \sin \vartheta_{2,L} = 0\,. \tag{15}$$

Thus, the appearance of an RCR for interaction of two plane unidentical SSW is more probable, the larger the ratio between the velocities of interacting waves, and conversely, at any value of this ratio not exceeding 1.117 for gases, an RCR cannot be implemented.

NOTATION

a, ratio of the velocity of a stronger shock wave to a weaker one ($a \geq 1$); D, shock wave velocity; c, speed of sound in a medium; p, pressure; q, total flow velocity in a coordinate system connected with the crossover point of the SSW; k, polytropic index of a gas; ρ, density; φ, impact angle of two SSW; ϑ, angle of rotation of the total flow velocity vector behind the shock wave front. Subscripts: H and L, upper and lower portions of flow with respect to SC; R, critical angle of the regular collision regime.

REFERENCES

1. R. Courant and K. O. Freidrichs, *Supersonic Flow and Shock Waves*, New York (1948).